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# ENHANCEMENT OF DEFORMATION MODELLING IN ENGINEERING AND GEOSCIENCES BY COMBINING DETERMINISTIC AND GENERALIZED GEOMETRICAL ANALYSES

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## ABSTRACT

Over the past ten years, the Engineering and Mining Surveys Research Group at the University of New Brunswick has developed a generalized method of geometrical analysis of deformation measurements and powerful software, FEMMA, for elastic and visco-elastic finite element modelling of the load-deformation relationship. The generalized method allows for a simultaneous integrated analysis of geodetic and geotechnical deformation measurements even if scattered in space and over time. By an iterative combination of the results of the geometrical analysis (displacement and strain fields) with the finite element analysis, a significant improvement in the understanding of the mechanism of deformation is obtained. The methodology has been successfully implemented in a number of engineering and geoscience projects. Two examples are given in this paper: a slope stability study in a coal mining area near Sparwood, B.C., and a dam deformation analysis at Mactaquac hydro-electric power generating station.

# INTRODUCTION

The monitoring and analysis of deformations are the subject of intensive studies by many professional groups which include, besides surveyors and geodesists, structural, mining, geotechnical, rock mechanics, and mechanical engineers, as well as geophysists. Safety, economical design of man-made structures, efficient production, environmental protection, and development of mitigative measures in case of natural disasters, require a good understanding of the mechanism of deformations and development methods for the deformation prediction. Even the most precise monitoring surveys will not fully serve their purpose if they are not properly evaluated and utilized in a global integrated analysis as a cooperative interdisciplinary effort. In deformation studies, among the most active international organizations is Working Group 6C of the International Federation of Surveyors (FIG). The Engineering and Mining Surveys Research Group at the University of New Brunswick (UNB) has been very involved in the works of FIG, and over the past 15 years the group has developed concepts of a generalized approach to an integrated analysis of deformation measurements. This paper gives a brief review of this approach, which combines geometrical analysis with physical interpretation of deformations for a better understanding of the mechanism of deformations. Two case studies illustrate the far methodology at its current stage. The research on its further enhancement is still in progress.

# CONCEPTS AND METHODOLOGY OF THE INTEGRATED ANALYSIS

The analysis of deformation measurements includes:

- geometrical analysis, which describes the geometrical status of the deformable body, its change in shape and dimensions, as well as rigid body movements (translations and rotations) of the whole deformable body with respect to a stable reference frame or of a block of the body with respect to other blocks, and
- physical interpretation which describes the state of internal stresses and the relationship between the causative effects (loads) and deformations.

As far as the geometrical analysis is concerned, the UNB Group, within the activities of FIG, has developed a so-called UNB Generalized Method of Deformation Analysis (Chen, 1983; Chrzanowski et al. 1986). This method is applicable to geometrical deformation analysis both in space and in time, including the detection of unstable reference points and the determination of strain components as well as absolute and relative rigid body motions. An important aspect of the generalized method is the possibility of utilizing in a simultaneous integrated analysis any type of geodetic and geotechnical observations, even if scattered in space and time, as long as approximate coordinates of all the observation points are known.

As far as the physical interpretation is concerned, it consists of: (a) a statistical (empirical) method, which analyzes through a regression analysis the correlations between observed deformations and observed loads (external and internal causes producing the deformation), and (b) a deterministic method, which utilizes information on the loads, properties of the materials, and physical laws governing the stress-strain relationship.

Once the load-deformation relationship is established, either by deterministic or by statistical regression analysis, the results may be used to develop prediction models. Through a comparison of predicted deformations with the results of the geometrical analysis of actual deformations, a better understanding of the mechanism of deformations is achieved. On the other hand, the prediction models supply information on expected deformations, facilitating the design of the monitoring scheme as well as the selection of the deformation model in the geometrical analysis. Thus, the expression *integrated analysis* means a determination of the deformation by combining all types of measurements, even if scattered in time and space, in the simultaneous geometrical analysis of the deformation and comparing it with the prediction model. If the differences are statistically significant, the explanation is sought and the prediction model is iteratively adjusted. Recently, a concept of a global integration has been developed at UNB (Chrzanowski et al., 1990) in which all three — the geometrical analysis of deformation and both methods of the physical interpretation — are combined for the final interpretation. The method still requires further elaboration, software development, and practical testing. In this presentation, only a combination of deterministic modelling with geometrical analysis is illustrated with two practical examples. This is preceded by a short description of the UNB Generalized Method for geometrical analysis and software FEMMA for deterministic modelling and prediction of deformations.

## GEOMETRICAL ANALYSIS OF DEFORMATIONS USING THE UNB GENERALIZED METHOD

The change in shape and dimensions of a 3-D deformable body is fully described if 6 strain components (3 normal and 3 shearing strains) and 3 differential rotations are determined at every point of the body. These deformation parameters can be calculated from the well-known strain-displacement relations if a displacement function representing the deformation of the object is known. Since, in practice, deformation surveys involve only discrete points, the displacement function must be approximated through some selected deformation model that fits the observed changes in coordinates (displacements), or any other types of observables, in the statistically best way. The displacement function may be determined, for example, through a polynomial approximation of the displacement field.

The displacement function can be expressed in matrix form in terms of a deformation model **Bc** as:

$$[1] \qquad \mathbf{d}(x, y, z, t\text{-}t_o) = (u,v,w)^T = \mathbf{B}(x, y, z, t\text{-}t_o)\, \mathbf{c}$$

where **d** is the displacement of a point (x,y,z) at time t with espect to a reference time $t_o$; u,v,and w are components of the displacement function in the x-, y-, and z- directions, respectively, **B** is the deformation matrix with its elements being some selected base functions; and **c** is the vector of unknown coefficients (deformation parameters).

A vector **Δl** of changes in any type of observations, for instance, changes in tilts, changes in observed strain or in observed distances, can always be expressed in terms of the displacement function. The functional relationships for any other types of observables and displacement functions are given in Chen (1983) and Chrzanowski et al. (1986). In matrix form, the relationship is written as:

$$[2] \qquad \mathbf{\Delta l} = \mathbf{A B \Delta l}\ \mathbf{c}$$

where **A** is the transformation matrix relating the observations to the displacement of points at which the observations are made, and **BΔl** is constructed from the above matrix **B**(x, y, z, t) and is related to the points included in the observables.

If redundant observations are made, the elements of the vector **c** and their variances and covariances are determined through least-squares approximation, and their statistical significance can be calculated. One tries to find the simplest possible displacement function that would fit the observations in the statistically best way. The search for the 'best' deformation model (displacement function) is based on either a priori knowledge of the expected deformations (for instance from a finite element analysis) or a qualitative analysis of the deformation trend and experience of the person involved in the analysis. In summary,the geometrical deformation analysis is done in 3 steps:

(1) A trend analysis in space and time domains and the selection of a few alternative deformation models which seem to match the trend and that make physical sense.

(2)A least-squares fitting of the model or models into the observation data and statistical testing of the models.

(3)A selection of the 'best' model that has as few coefficients as possible with as high a significance as possible (preferably all the coefficients should be significant at probabilities greater than 95%).

The results of the geometrical analysis serve as a basis for a preliminary qualitative physical interpretation that helps to enhance the deterministic model .

## DETERMINISTIC MODELLING OF DEFORMATIONS USING 'FEMMA'

As far as the deterministic modelling of deformations is concerned, the finite element, boundary element, and finite differences numerical methods, which were domains of mechanical, civil, and rock mechanics engineers, become tools of surveying engineers. A powerful software FEMMA (Szostak-Chrzanowski and Chrzanowski, 1991) for 2-D and 3-D finite elements elastic, visco-elastic, and heat transfer analyses of deformations has been developed at UNB and is being applied in various fields of engineering and geoscience projects to the integrated analyses of deformations.

The software FEMMA has been developed in two main versions, FEMMA 2.0 and FEMMA 3.0, for two- and three-dimensional linear elastic and visco-elastic analyses, respectively. The two-dimensional version has been adapted for PC computers, while the three-dimensional version is still usable only on mainframe computers. Both versions have an option of using either metre or kilometre as the basic linear unit depending on the scale of the project. All versions of FEMMA are supported by the automatic mesh generation software MESHGEN 2.0 and 3.0.

The main characteristics of FEMMA are:

- linear elastic or visco-elastic analyses;
- thermal analysis: steady state heat-transfer;
- analysis with isotropic and anisotropic materials (the direction of anisotropy does not have to coincide with the axes of the coordinate system);
- optional use of either two-nodal (bars), or three-nodal (triangular), or four-nodal elements in the two-dimensional analyses and use of eight nodal ('bricks') elements in three-dimensional problems;
- in the visco-elastic analyses either the Kelvin model as given in Zienkiewicz et al. (1968b), or the general power law for creep (Beddoes et al., 1989) can be used (any other model can be added);
- modelling of discontinuities using either the split node technique (Melosh and Raefsky, 1981) or anisotropic elements;
- use of no-tension (Zienkiewicz et al., 1968a) and Hoek-Brown criteria (Hoek and Brown, 1980) in rock deformation studies;
- use of initial strain, initial stress, and calculated initial thermal strain;
- in two-dimensional analyses, options for either plane stress or plane strain cases are available;

- any number of material types (limited only by the number of elements);
- banded format of the total stiffness matrix (to decrease the dimensions of the total stiffness matrix);
- modular structure of the program for easy introduction of changes and adaptations for various applications.

The output of the program gives:
- absolute and relative displacement vectors at the nodal points (x, y, z components, total displacements and their orientations),
- total strain vector in each element with elastic and creep components,
- stress vector in each element,
- principal stresses and their orientation,
- list of elements indicated by chosen criteria,
- temperatures at each element.

Among the strong points of FEMMA are its versatility to perform either 2-D or 3-D modelling, its modular structure that allows for easy expansion and/or modification to model any type of visco-elastic behaviour, and its flexibility to determine relative movements along discontinuities (e.g. geological faults) using either the split node technique or a method based on introducing anisotropic elements within the discontinuity zone. In deformation studies of the earth's crust, FEMMA permists the calculation of changes in gravity due to tectonic plate movements and resulting changes in geodetic heights.

## CASE STUDIES

### Integrated Analysis of Ground Subsidence in Sparwood, B.C.

The UNB Generalized Method was applied in an integrated analysis of survey data collected over an underground coal mining operation in rugged mountainous terrain of western Canada near Sparwood, British Columbia. The purpose of the survey was to monitor ground movements caused by extraction of a 200 m by 700 m panel of a 12 m thick and steeply inclined coal seam (Fig.1). Three types of observations in three dimensions were used in the integrated analysis of ground subsidence above the panel: changes in coordinates of 15 points determined by terrestrial geodetic methods, changes in coordinates of 29 points calculated from aerial photogrammetric surveys, and changes in ground tilts at three stations obtained from remotely controlled biaxial tiltmeters (Chrzanowski and Fisekci, 1982). Displacements of up to 2 m over a three-year monitoring period were observed at some points on the slope. Following the steps of the UNB Generalized Method of geometrical analysis, several possible deformation models (displacement functions) were fitted to the observations. Figure 2 shows a computer generated graphical display of ground subsidence calculated from the model.

The extraction of the coal panel, in addition to the described deformations, produced surface cavings (Fig.1) above the upper edge of the panel near the outcrop and long cracks near the mountain ridge. Also some discontinuity in the coal seem was identified in the underground development workings along the lower edge of the extracted coal panel. The cracks on the surface could not be readily explained due to very limited knowledge of the geology and tectonics of the area. A fault in the rock strata connecting the area of the surface cracks with the approximately mapped discontinuity at the level of the mine workings could be a possible explanation. Confirmation of the existence of the fault was of significant importance for the safety of the mine operation. Consequently, deterministic modelling of the rock strata deformation was performed based on the finite element analysis using the software FEMMA and an iterative method of ground subsidence prediction developed earlier by Szostak-Chrzanowski (1988).

Two iterative analyses were made: one without introducing the suspected fault into the FEM model, and the second with the fault zone represented by a string of elements between the developed crack on the surface and the mapped discontinuity in the rock masses in the underground workings (Fig.3). The elements in the fault zone were modelled as anisotropic material with the Young modulus $E = 0$ in the direction perpendicular to the fault plane. The second solution gave a good agreement with the result of the geometrical analysis (Fig.3) thus confirming the existence of the suspected rock strata discontinuity.

## Integrated Deformation Analysis of the Powerhouse at the Mactaquac Power Dam

The UNB Generalized Method was applied to an integrated analysis of deformations of a concrete gravity dam and powerhouse structures at the Mactaquac hydro-electric power generating station in New Brunswick, Canada (Chrzanowski et al., 1991). In the mid-1970s, abnormal deformations in both structures were noticed in the form of cracks, opening of vertical construction joints (at the rate of up to 3 mm/year in the powerhouse), and leakage through horizontal construction joints in the intake.

Numerous theories were put forward by various consultants to explain the abnormal structural deformations. At first, the theories included regional and local rock movements, transfer of water load through the penstocks to the powerhouse, effects of alkali-aggregate reactivity in concrete, residual stress, and squeeze and/or rebound of the foundation.

To better understand the mechanism and causes of deformation, an extensive monitoring scheme was established in the early 1980s which includes precision geodetic surveys (horizontal and vertical) and measurements with geotechnical instruments such as multi-rod borehole extensometers, invar tape and rod extensometers, suspended and inverted plumb-lines, strain gauges, tiltmeters, and various joint meters and tell-tales across the joint openings and structural cracks. Since 1986, GPS surveys have been added to the monitoring scheme to check on regional stability.

A trend analysis (plot of observed changes vs. time) of all observations was performed indicating that the deformations were fairly linear in time after compensating the observations for seasonal (thermal and water level) periodic variations. Therefore, the average rates of observation change could be taken for the spatial trend analysis. Figure 4 gives an example of rates of deformation (mm/year) derived from a sample of measurements taken at frequent intervals (most of the observations were taken bi-weekly) over a period of three years in one upstream-downstream cross section of the powerhouse.

Using the UNB Generalized Method, several different functions (full or partial polynomials) were attempted in fitting the observation data, including relative rotations and translations between the structural blocks and different deformations in each block. After eliminating all the statistically insignificant coefficients of the selected displacement functions, the final model appeared to be quite simple, with the bedrock accepted as stable, and the whole structure undergoing the same deformation described by the displacement (rates per year) functions:

$$u(x, z) = a_1 x + a_2 xz + a_3 x^2 \quad \text{and} \quad w(x, z) = bz$$

in which u(x, z) and w(x, z) describe the horizontal and vertical components of the displacement field, respectively. The $a_i$ and $b_i$ coefficients have been determined through a least-squares fitting to the deformations.

Figure 5 gives a graphical display of the displacement field (rates per year) in the investigated cross section. The displacements and strain parameters derived from the displacement function clearly indicated a volumetric expansion of the whole structure. The results of the integrated analysis supported the postulated earlier hypothesis about possible swelling of concrete due to the alkali-aggregate reaction. To further corroborate the finding, an independent (not a simple back analysis) finite element analysis of the deformation was performed using software FEMMA. In the FEM analysis, the assumed growth of the concrete was introduced as an initial strain rate, with the basic rate for unconstrained concrete blocks in the powerhouse being 0.2 mm/m/year as deduced from the geometrical analysis. Because of a non-homogeneous distribution of the steel reinforcement in the structures, the input strains in the FEM analysis were differentiated from one element to another by reducing the basic rate proportionally to the approximate percentage content of reinforcement steel in the concrete. Figure 6 shows the FEM mesh and sample nodal displacements, which compare very well with the displacements in the powerhouse from the geometrical analysis. Thus, in spite of the complexity involved with the various sources and quality of data, the various locations for observation, and the behaviour of the structure, an integrated analysis of the deformations of the powerhouse was achieved and this strengthened the assumption that the growth of concrete is the major source of the deformation.

## CONCLUSIONS AND RECOMMENDATIONS

Proper analysis and interpretation of deformations are complex processes, and they should always be given to a person who has a good understanding of the mechanics of deformable bodies and of the newest methods of analysis. Even the most precise monitoring surveys may lead to wrong and, sometimes, dangerous conclusions if they are not properly analysed and interpreted. The UNB Generalized Method of geometrical analysis of deformations and the powerful software FEMMA for finite element analysis , if used together, are powerful tools for integrated analysis and prediction of deformations.

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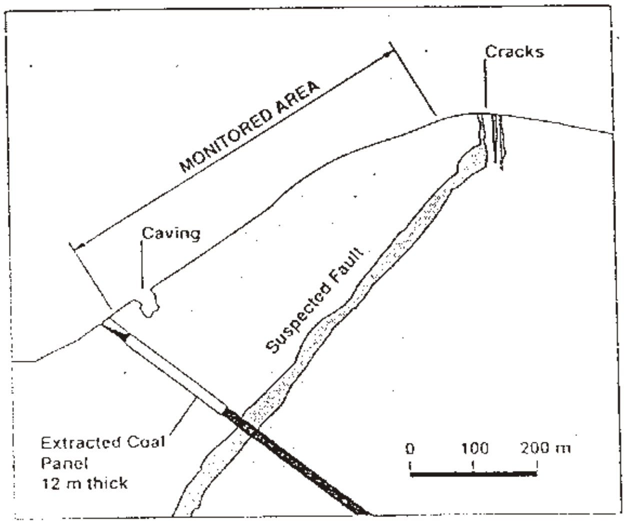

Fig. 1. Coal mine in Sparwood, B.C., vertical cross-section

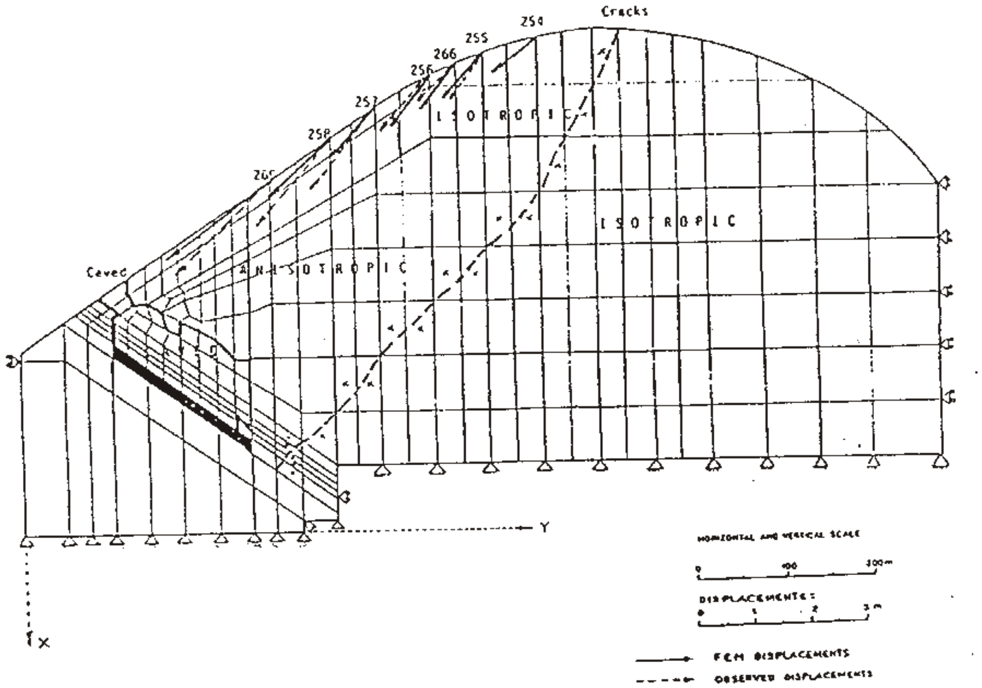

Fig. 2. Comparison of FEM and observed displacements (FEM model with fault)

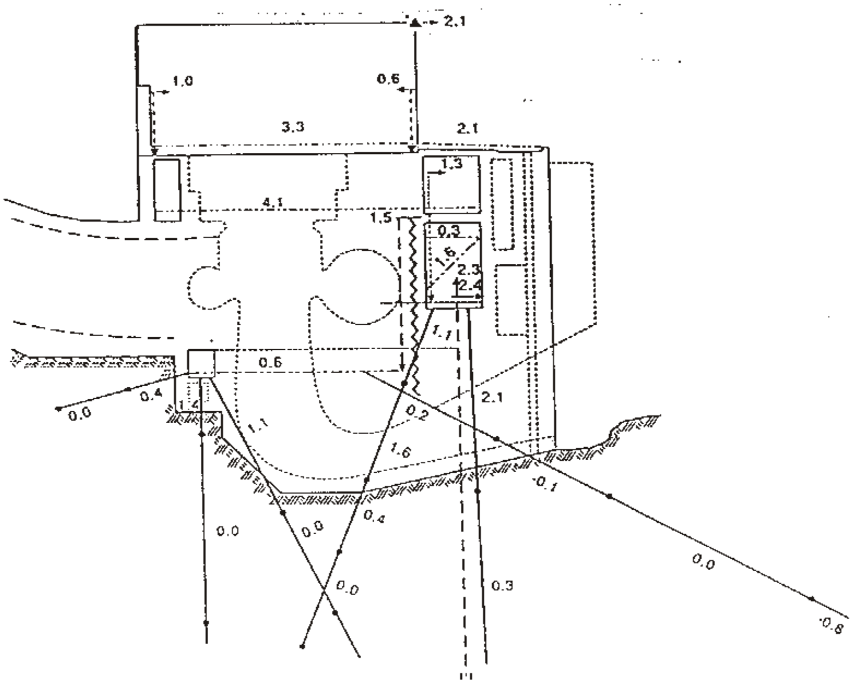

Fig.3. Observed rates of deformations [mm/a] in a vertical cross-section of Powerhouse at Mactaquac

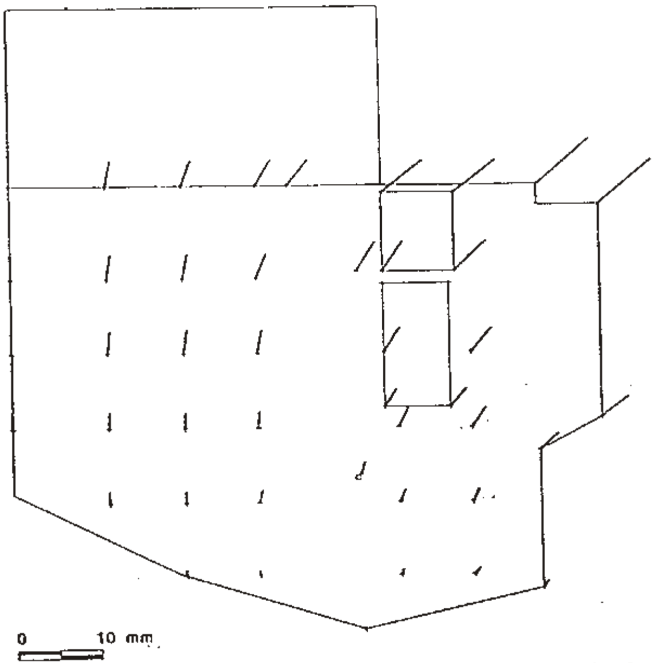

Fig. 4. Displacement field obtained from the generalized geometrical analysis

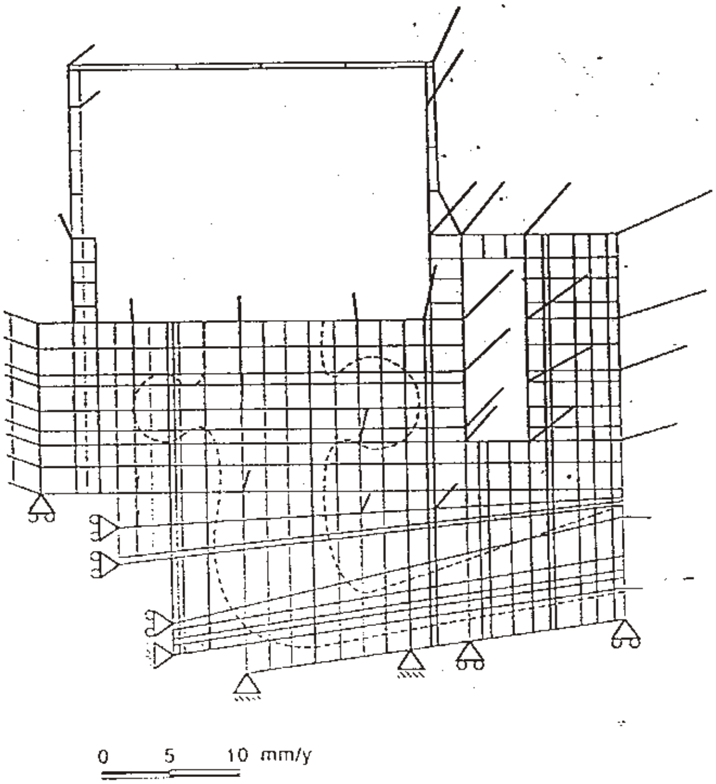

Fig.5. FEMMA displacement field